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TRANSIENT MAGNETOHYDRODYNAMIC FLOW IN AN ANNULAR CHANNEL

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ABSTRACT

Unsteady parallel flow of an electrically conducting viscous incompressible fluid in an annular channel in the presence of a radial transverse magnetic field is considered. Assuming the fluid to be at rest at the initial moment, the velocity distribution and magnetic field components are obtained in terms of Bessel and Lommel functions and in the form of convolution integrals taking the longitudinal pressure as an arbitrary function of time. Further, taking a step function for the pressure gradient, these expressions are integrated. The influence of the magnetic field on the annular flow is investigated. Some numerical examples are given.

TRANSIENT MAGNETOHYDRODYNAMIC FLOW IN AN ANNULAR CHANNEL

M. N. L. Narasimhan

Introduction.

In a previous paper (Narasimhan 1963) we have investigated a pulsating flow in an annular channel with a radial transverse magnetic field. The problem of unsteady magnetohydrodynamic flow in a rectangular duct has been solved by Lundgren, Atabek and Chang (1961). The purpose of the present paper is to obtain a transient solution for the problem of unsteady flow in an annular channel with an impressed radial transverse magnetic field. We consider an infinitely long annular channel of inner radius a and outer radius b, shown in Figure 1.

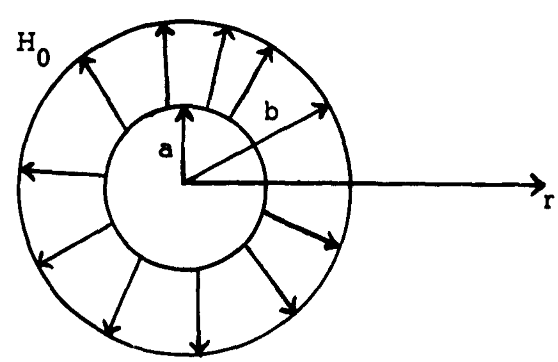

Figure 1. Annular channel with impressed radial field.

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We consider a radial magnetic field $H_0 = \frac{\omega}{r}$, where ω is a constant, impressed across the channel. In practice, it is possible to obtain an approximation to the desired field as explained by Globe (1959).

Governing equations.

The non-steady flow of an electrically conducting incompressible fluid (mercury for instance) in the presence of a magnetic field is governed by the following equations (in m.k.s. units)

$$\nabla \times \vec{H} = \vec{J} \tag{1}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \tag{2}$$

$$\nabla \cdot \vec{H} = 0 \tag{3}$$

$$\vec{J} = \sigma (\vec{E} + \mu \vec{V} \times \vec{H}) \tag{4}$$

$$\nabla \cdot \vec{V} = 0 \tag{5}$$

$$\rho \left[\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \right] = -\nabla p + \rho \nu \nabla^2 \vec{V} + \mu \vec{J} \times \vec{H} \quad . \tag{6}$$

The first four equations are Maxwell's equations, which govern the electromagnetic field, Eq. (5) and (6) are respectively the equation of continuity of fluid and the equation of motion. In the above equations we have neglected the displacement current and assumed that the permeability and dietectric constant are the same as in a vacuum. Also we have neglected the free charge. We use cylindrical coordinates (r, θ, z) and make the following assumptions in the manner of Globe (1959):

(1) $\frac{\partial}{\partial \theta} = 0$, on account of axial symmetry.

(2) $v_r = v_\theta = 0$.

(3) We assume that the applied field $H_0 = \frac{\omega}{r}$ fixes the normal component of the magnetic field at $r = a$ and $r = b$ for all values of z, and that this is the only field impressed. Several consequences follow from this assumption.

(a) Now from (1) and (4) we have

$$j_z = \sigma [E_z + \mu(\vec{V} \times \vec{H})_z] = \frac{\partial H_\theta}{\partial r} + \frac{H_\theta}{r} \qquad (a.1)$$

which must vanish since E_z can arise only from an applied field $\vec{E}$ or free charges in the flow, neither of which exists, and because $(\vec{V}\times\vec{H})_z$ must vanish, since $\vec{V}$ has a z-component only. Again from (1) and (4) we have

$$j_r = \sigma [E_r + \mu(\vec{V} \times \vec{H})_r] = -\mu\sigma V_z H_\theta = \frac{-\partial H_\theta}{\partial z} \ . \qquad (a.2)$$

Thus we have

$$j_z = 0 = \frac{\partial}{\partial r}(rH_\theta) \text{ or } H_\theta = \frac{f(z)}{r} \ . \qquad (a.3)$$

Hence from (a.2) and (a.3) we get

$$V_z = \frac{1}{\mu\sigma} \frac{f'(z)}{f(z)} \ , \qquad (a.4)$$

a function of z alone.

From (5), and assumptions (2) and (a.4), we obtain

$$\frac{d}{dz}\left(\frac{f'(z)}{f(z)}\right) = 0 \quad ,$$

whose solution has the form $f(z) = Ae^{Bz}$, where A and B are constants. This would make V_z a constant and since V_z has to vanish at the boundaries, this constant would be zero. Thus the only possibility we have to consider is that $f(z) = 0$, so that from (a.3), $H_\theta = 0$ and hence j_r also vanish everywhere in the channel.

(b) From (3), we have

$$\frac{\partial H_r}{\partial r} + \frac{H_r}{r} + \frac{\partial H_z}{\partial z} = 0 \quad ,$$

whose solution satisfying the boundary conditions can be obtained by putting

$$\frac{\partial H_z}{\partial z} = -2F \quad ;$$

therefore

$$\frac{\partial H_r}{\partial r} + \frac{H_r}{r} = 2F \quad ,$$

from which it follows

$$H_r = Fr + \frac{G}{r} \quad ,$$

where F and G are constants at any instant of time.

Since the radial magnetic field is to remain at its prescribed values $\frac{\omega}{a}$ and $\frac{\omega}{b}$ at $r = a$ and $r = b$, respectively, for all values of z, we obtain

$$\frac{\omega}{a} = Fa + \frac{G}{a}, \; \frac{\omega}{b} = Fb + \frac{G}{b},$$

which shows that $F = 0$ and $G = \omega$.

Therefore

$$\frac{\partial H_z}{\partial z} = 0 \text{ and } \frac{\partial}{\partial r}(rH_r) = 0 \text{ or } rH_r = \text{constant.}$$

The radial field must then be equal to the impressed field and is unaffected by the flow and is independent of z .

Now we eliminate $\vec{J}$ and $\vec{E}$ from (1), (2) and (4) and obtain the following equation

$$\frac{\partial \vec{H}}{\partial t} - (\vec{H} \cdot \nabla)\vec{V} + (\vec{V} \cdot \nabla)\vec{H} = \frac{1}{\mu\sigma} \nabla^2 \vec{H} \quad . \tag{9}$$

With the above assumptions, Equations (6) and (9) reduce to three in number:

$$\frac{\partial p}{\partial r} + \mu H_z \frac{\partial H_z}{\partial r} = 0 \quad , \tag{10}$$

$$\rho \frac{\partial v_z}{\partial t} - \frac{\mu\omega}{r} \frac{\partial H_z}{\partial r} = -\frac{\partial p}{\partial z} + \rho\nu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right) \quad , \tag{11}$$

$$\mu\sigma \frac{\partial H_z}{\partial t} - \frac{\mu\omega\sigma}{r} \frac{\partial v_z}{\partial r} = \frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} \quad . \tag{12}$$

H_z and v_z are functions of r and t only. It follows from (11) that $\frac{\partial p}{\partial z}$ must be independent of z . By differentiating (10) with respect to z, it can be seen that $\frac{\partial p}{\partial z}$ is independent of r also. We may therefore write

$$\frac{\partial p}{\partial z} = - P(t) \quad .$$

Once H_z is determined, the variation of p across the channel may be found by integrating (10):

$$p(r,z,t) + \frac{\mu}{2} H_z^2 = - P(t)\, z \quad .$$

Now (11) and (12) can be rewritten as:

$$\rho \frac{\partial v_z}{\partial t} - \frac{\mu\omega}{r} \frac{\partial H_z}{\partial r} = P(t) + \rho\nu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right) \tag{13}$$

$$\mu\sigma \frac{\partial H_z}{\partial t} - \frac{\mu\omega\sigma}{r} \frac{\partial v_z}{\partial r} = \frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} \quad . \tag{14}$$

The initial conditions are:

$$v_z(r,t) = 0 \ , \qquad \text{at } t = 0 \ . \tag{15}$$

$$H_z(r,t) = 0 \ , \qquad \text{at } t = 0 \ . \tag{16}$$

The boundary conditions of the problem are:

$$v_z(r,t) = 0 \qquad \text{at } r = a \tag{17}$$

$$v_z(r,t) = 0 \qquad \text{at } r = b \tag{18}$$

$$H_z(r,t) = 0 \qquad \text{at } r = b \tag{19}$$

$$\frac{\partial H_z}{\partial r}(r,t) = 0 \qquad \text{at } r = b \ . \tag{20}$$

(17) and (18) are the no slip conditions.

(19) follows from the fact that $\vec{J}$ has a θ-component only so that the currents in the annular channel are like those in an infinite solenoid. These currents produce no field for $r > b$ and since there is no impressed field in the z-direction, continuity of the tangential component of $\vec{H}$ requires (19) to be true. (20) is justified as follows:

$$j_\theta = -\frac{\partial H_z}{\partial r} \ .$$

but

$$j_\theta = \sigma\mu(\vec{V} \times \vec{H})_\theta$$

and V must vanish at $r = b$. Hence j_θ must also vanish there and $\frac{\partial H_z}{\partial r}$ too.

Now we non-dimensionalize the equations (13) to (20) by introducing the following dimensionless quantities:

Let
$$\lambda = \frac{r}{a}, \qquad \tau = \frac{t\nu}{a^2(\mu\sigma\nu)^{\frac{1}{2}}},$$

$$V = \frac{v_z}{(\nu/a)}, \qquad H = \frac{H_z}{(\nu/a)(\sigma\rho\nu)^{\frac{1}{2}}},$$

$$\phi(\tau) = -a^3\rho^{-1}\nu^{-2}\frac{\partial p}{\partial z} = a^3\rho^{-1}\nu^{-2}P(t)$$

$$\beta^2 = \frac{\mu^2\omega^2\sigma}{\rho\nu}, \qquad \gamma = (\mu\sigma\nu)^{\frac{1}{2}} \text{ and let } \frac{b}{a} = \delta;$$

β is a form of the Hartmann number.

When these non-dimensional quantities are introduced into the equations (13) to (20), the latter take the form:

$$\frac{1}{\gamma}\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial\lambda^2} + \frac{1}{\lambda}\frac{\partial V}{\partial\lambda} + \frac{\beta}{\lambda}\frac{\partial H}{\partial\lambda} + \phi(\tau), \tag{21}$$

$$\gamma\frac{\partial H}{\partial t} = \frac{\partial^2 H}{\partial\lambda^2} + \frac{1}{\lambda}\frac{\partial H}{\partial\lambda} + \frac{\beta}{\lambda}\frac{\partial V}{\partial\lambda}, \tag{22}$$

with the conditions:

$$V(\lambda,\tau) = 0, \qquad \text{at } \tau = 0 \tag{23}$$

$$H(\lambda,\tau) = 0, \qquad \text{at } \tau = 0 \tag{24}$$

$$V(1,\tau) = 0, \tag{25}$$

$$V(\delta,\tau) = 0, \tag{26}$$

$$H(\delta,\tau) = 0 \ , \tag{27}$$

$$\frac{\partial H}{\partial \lambda}(\lambda,\tau) = 0 \qquad \text{at } \lambda = \delta \ . \tag{28}$$

The solution.

First let us eliminate τ from the equations by taking a Laplace transform with respect to τ. Let s be the transformed parameter and $\bar{V}$, $\bar{H}$ and $\bar{\phi}$ denote transformed variables. Thus our problem reduces to:

$$\frac{\partial^2 \bar{V}}{\partial \lambda^2} + \frac{1}{\lambda}\frac{\partial \bar{V}}{\partial \lambda} + \frac{\beta}{\lambda}\frac{\partial \bar{H}}{\partial \lambda} = \frac{s}{\gamma}\bar{V} - \bar{\phi} \ , \tag{29}$$

$$\frac{\partial^2 \bar{H}}{\partial \lambda^2} + \frac{1}{\lambda}\frac{\partial \bar{H}}{\partial \lambda} + \frac{\beta}{\lambda}\frac{\partial \bar{V}}{\partial \lambda} = \gamma s \bar{H} \ , \tag{30}$$

with the conditions:

$$\bar{V}(1,s) = 0 \ , \tag{31}$$

$$\bar{V}(\delta,s) = 0 \ , \tag{32}$$

$$\bar{H}(\delta,s) = 0 \ , \tag{33}$$

$$\frac{\partial \bar{H}}{\partial \lambda}(\lambda,s) = 0 \qquad \text{at } \lambda = \delta \ . \tag{34}$$

The solution of this system of equations for general values of $\gamma = (\mu\sigma\nu)^{\frac{1}{2}}$ runs into difficulties. Moreover, in actual physical situations, it will be sufficient to solve the system for $\gamma << 1$, since for most of the incompressible, electrically conducting viscous fluids on the surface of the earth, $\gamma << 1$.

For instance (Lundgren, Atabek, Chang 1961):

Table 1

Fluid	$\gamma = (\mu\sigma\nu)^{\frac{1}{2}}$
Hg (20°C)	3.56×10^{-4}
Na (500°C)	1.55×10^{-3}
Pb (500°C)	0.47×10^{-3}

Hence we shall assume $\gamma << 1$ and $s \sim \gamma$. The equations (29) and (30), after neglecting terms of order γ^2, reduce to:

$$\frac{\partial^2 \bar{V}}{\partial \lambda^2} + \frac{1}{\lambda} \frac{\partial \bar{V}}{\partial \lambda} + \frac{\beta}{\lambda} \frac{\partial \bar{H}}{\partial \lambda} = \frac{s}{\gamma} \bar{V} - \bar{\phi} \qquad (35)$$

and
$$\frac{\partial^2 \bar{H}}{\partial \lambda^2} + \frac{1}{\lambda} \frac{\partial \bar{H}}{\partial \lambda} + \frac{\beta}{\lambda} \frac{\partial \bar{V}}{\partial \lambda} = 0 \ . \qquad (36)$$

From (36) after integrating once and using (32) and (34) we obtain

$$\frac{\partial \bar{H}}{\partial \lambda} = - \beta \frac{\bar{V}}{\lambda} \ . \qquad (37)$$

Substitution of (37) into (35) yields the following non-homogeneous modified Bessel equation:

$$\lambda^2 \frac{\partial^2 \bar{V}}{\partial \lambda^2} + \lambda \frac{\partial \bar{V}}{\partial \lambda} - (\lambda^2 \frac{s}{\gamma} + \beta^2) \bar{V} = - \lambda^2 \bar{\phi} \ . \qquad (38)$$

The general solution of (38) is therefore of the form

$$\bar{V}(\lambda, s) = A_1 I_\beta(\lambda\sqrt{s/\gamma}) + A_2 K_\beta(\lambda\sqrt{s/\gamma}) + \frac{\gamma}{s}\bar{\phi}(s)\, s_{1,\beta}(i\lambda\sqrt{s/\gamma}) \quad , \tag{39}$$

where the functions I_β, K_β are modified Bessel functions of the first and second kind respectively and of order β, and $s_{1,\beta}$ is a Lommel function (Watson, 1944) which occurs as a particular integral of (38) . A_1 and A_2 are arbitrary constants. This particular integral can be written either as ascending or descending power series in the argument (Watson 1944) according as $1 \pm \beta \neq -(2p-1)$ or $1 \pm \beta = -(2p-1)$, where p is a positive integer. For the sake of simplicity we shall demonstrate the solution when $1 \pm \beta \neq -(2p-1)$. This condition is satisfied when $\beta =$ an odd positive integer or zero. Thus we shall demonstrate the solution when β, the Hartmann number is an odd positive integer or zero.

Hence in this case we obtain the complete solution of (38) as:

$$\bar{V}(\lambda, s) = \frac{\bar{\phi}\gamma}{s}\left[\frac{\left\{\begin{array}{l} s_{1,\beta}(i\delta\sqrt{s/\gamma})\,\{K_\beta(\sqrt{s/\gamma})\, I_\beta(\lambda\sqrt{s/\gamma}) - I_\beta(\sqrt{s/\gamma})\, K_\beta(\lambda\sqrt{s/\gamma})\} \\ - s_{1,\beta}(i\sqrt{s/\gamma})\,\{K_\beta(\delta\sqrt{s/\gamma})\, I_\beta(\lambda\sqrt{s/\gamma}) - I_\beta(\delta\sqrt{s/\gamma})\, K_\beta(\lambda\sqrt{s/\gamma})\} \end{array}\right\}}{I_\beta(\sqrt{s/\gamma})\, K_\beta(\delta\sqrt{s/\gamma}) - I_\beta(\delta\sqrt{s/\gamma})\, K_\beta(\sqrt{s/\gamma})} + s_{1,\beta}(i\lambda\sqrt{s/\gamma})\right] \tag{40}$$

where

$$s_{\mu,\nu}(z) = \frac{z^{\mu+1}}{(\mu+1)^2-\nu^2} - \frac{z^{\mu+3}}{\{(\mu+1)^2-\nu^2\}\{(\mu+3)^2-\nu^2\}} + \ldots \; ; \tag{41}$$

and $\mu \pm \nu \neq -(2p-1)$, p being a positive integer.

Now taking the inverse Laplace transform of (40) and using the convolution theorem (Carslaw and Jaeger 1941) we obtain

$$V(\lambda,\tau) = \sum_{n=1}^{\infty} A_n \int_0^{\tau} \phi(\tau-\xi)\exp(-\gamma\,\alpha_n^2\,\xi)\,d\xi \quad , \tag{42}$$

where

$$A_n = \frac{\pi\gamma\, J_\beta(\alpha_n)\, J_\beta(\alpha_n\delta)}{J_\beta^2(\alpha_n\delta) - J_\beta^2(\alpha_n)} \left[\begin{array}{l} s_{1,\beta}(\alpha_n\delta)\{J_\beta(\alpha_n)\,Y_\beta(\alpha_n\lambda) - J_\beta(\alpha_n\lambda)\,Y_\beta(\alpha_n)\} \\ \\ -s_{1,\beta}(\alpha_n)\,\{J_\beta(\alpha_n\delta)\,Y_\beta(\alpha_n\lambda) - J_\beta(\alpha_n\lambda)\,Y_\beta(\alpha_n\delta)\} \end{array} \right], \tag{43}$$

J_β and Y_β being Bessel functions of the first and kind respectively and α_n's $(n = 1, 2, \ldots \infty)$ are the roots of the equation

$$J_\beta(\alpha)\,Y_\beta(\alpha\delta) - J_\beta(\alpha\delta)\,Y_\beta(\alpha) = 0 \quad , \tag{44}$$

β being a positive odd integer. These roots α_n's, $n = 1, 2, \ldots \infty$ are all known to be real and simple since δ and β are real. [Carslaw, Conduction of heat (London 1922) p. 128].

To integrate (42) we have to specify time dependence of the pressure gradient.

Step function pressure gradient.

Let us take $\phi(\tau)$ as a step function. That is, let

$$\phi(\tau) = \begin{cases} 0 & \text{for } \tau < 0 \\ \phi_0 & \text{for } \tau > 0 \end{cases} \tag{45}$$

Then after integration, Equation (42) gives us

$$V(\lambda,\tau) = \pi\phi_0 \sum_{n=1}^{\infty} \frac{J_\beta(\alpha_n) J_\beta(\alpha_n\delta)}{\alpha_n^2[J_\beta^2(\alpha_n\delta) - J_\beta^2(\alpha_n)]} \begin{bmatrix} s_{1,\beta}(\alpha_n\delta)\{J_\beta(\alpha_n)Y_\beta(\alpha_n\lambda) - J_\beta(\alpha_n\lambda)Y_\beta(\alpha_n)\} \\ -s_{1,\beta}(\alpha_n)\{J_\beta(\alpha_n\delta)Y_\beta(\alpha_n\lambda) - J_\beta(\alpha_n\lambda)Y_\beta(\alpha_n\delta)\} \end{bmatrix} \left(1 - e^{-\gamma\alpha_n^2\tau}\right). \tag{46}$$

By taking the inverse transform of (37) and using (26) and (28) we have

$$\frac{\partial H}{\partial \lambda} = -\beta \frac{V(\lambda,\tau)}{\lambda} . \tag{47}$$

If we now substitute (46) into (47) and integrate, we obtain after using (27):

$$H(\lambda,\tau) = \pi\phi_0 \sum_{n=1}^{\infty} \frac{J_\beta(\alpha_n) J_\beta(\alpha_n\delta)}{\alpha_n^2[J_\beta^2(\alpha_n\delta) - J_\beta^2(\alpha_n)]} \begin{bmatrix} s_{1,\beta}(\alpha_n\delta)\{J_\beta(\alpha_n)[w_n(\lambda) - w_n(\delta)] \\ \qquad -Y_\beta(\alpha_n)[u_n(\lambda) - u_n(\delta)]\} \\ -s_{1,\beta}(\alpha_n)\{J_\beta(\alpha_n\delta)[w_n(\lambda) - w_n(\delta)] \\ \qquad -Y_\beta(\alpha_n\delta)[u_n(\lambda) - u_n(\delta)]\} \end{bmatrix} \left(1 - e^{-\gamma\alpha_n^2\tau}\right), \tag{48}$$

where

$$u_n(\lambda) - u_n(\delta) = -\beta \int_\delta^\lambda \frac{J_\beta(\lambda \alpha_n)}{\lambda} d\lambda$$

$$w_n(\lambda) - w_n(\delta) = -\beta \int_\delta^\lambda \frac{Y_\beta(\lambda \alpha_n)}{\lambda} d\lambda \quad .$$

Thus (46) and (48) give the velocity field and magnetic field respectively.

Discussion of the solution.

The expressions (46) and (48) for V and H contain a time-independent part and a time-dependent part. The first parts, namely the time-independent parts represent the fully developed values of V and H, respectively.

In the time-dependent parts which contain exponential terms, we find that starting from rest V and H grow to their developed values without ever exceeding them. Since α_n's are all real, it is found that periodic or even partially periodic flows are impossible with the above choice of step function pressure gradient unlike the case of a similar type of flow in a rectangular duct with a transverse magnetic field (Lundgren, Atabek and Chang 1961). Thus it is found that with a suddenly applied pressure gradient the flow of conducting incompressible fluids under a radial transverse magnetic field is damped and partially periodic flows are impossible. We have calculated numerically the velocity distribution when the radius ratio of the annulus $\frac{b}{a} = 2$. The hydrodynamic and hydromagnetic cases corresponding to the values of the Hartmann number $\beta = 0$ and $\beta = 5$ have been obtained for purposes of comparison. The roots α_n's of the equation

$$J_\beta(\alpha)\, Y_\beta(\alpha\delta) - J_\beta(\alpha\delta)\, Y_\beta(\alpha) = 0 \; ; \qquad \delta = \frac{b}{a} = 2 \quad ,$$

have been calculated by using the formula for α_n's (Gray and Mathews 1895):

$$\alpha_n = k + \frac{p}{k} + \frac{q - p^2}{k^3} + \frac{r - 4pq + 2p^3}{k^5} + \ldots \quad ,$$

where

$$K = \frac{n\Pi}{\delta} \;, \quad p = \frac{m-1}{8\delta} \;, \quad q = \frac{4(m-1)(m-25)(\delta^3 - 1)}{3(8\delta)^3 (\delta - 1)} \quad ,$$

$$r = \frac{32(m-1)(m^2 - 114m + 1073)(\delta^5 - 1)}{5(8\delta)^5 (\delta - 1)} \quad , \qquad m = 4\beta^2 \quad ;$$

the first ten roots are tabulated for $\beta = 0$ and $\beta = 5$ in table 2 .

The Lommel function $s_{1,\beta}(x)$ for $\beta = 0$ reduces to the well known relation (Watson 1944),

$$s_{1,0}(x) = 1 - J_0(x) \quad ,$$

where $J_0(x)$ is the Bessel function of the first kind and order zero and $s_{1,0}(x)$ has been computed thus. The Lommel function $s_{1,\beta}(x)$ for $\beta = 5$ has been computed from equation (41) and for large arguments x the asymptotic relation (Watson 1944)

$$s_{\mu,\nu}(x) = x^{\mu-1}\left[1 - \frac{(\mu-1)^2 - \nu^2}{x^2} + \frac{\{(\mu-1)^2 - \nu^2\}\{(\mu-3)^2 - \nu^2\}}{x^4} \ldots\right]$$

has been used.

The results of the computation are given by the velocity profiles in Figure 2 and Figure 3, obtained as functions of the radial coordinate λ. In Figure 2 the velocity profiles for the hydrodynamic flow $(\beta = 0)$, have been calculated when $\gamma\tau = 0.1,\ 0.25,\ 0.5,\ 1.0$ and ∞. In Figure 3, the velocity profiles for the hydromagnetic flow $(\beta = 5)$, have been calculated when $\gamma\tau = 0.1,\ 0.5$ and ∞. From these figures, it is found that the effect of the magnetic field is to flatten the velocity distribution and to shorten the development time. In Figure 3, it is clearly seen that the flow development time for the hydromagnetic flow is very much less compared to the development time for the hydrodynamic flow.

Summary

In this paper the unsteady flow of a conducting fluid through an annular channel under a radial transverse magnetic field and a suddenly applied pressure gradient has been considered. For most conducting incompressible fluids for which $\gamma = (\mu\sigma\nu)^{\frac{1}{2}} << 1$, the response of the fluid is damped and partially periodic flows are found to be impossible. The effect of the magnetic field on the fluid flow is found to flatten the velocity distribution and to shorten the flow development time.

Table 2

The first ten roots α_n's of the Equation: $J_\beta(\alpha)\, Y_\beta(\alpha\delta) - J_\beta(\alpha\delta)\, Y_\beta(\alpha) = 0$ for $\beta = 0,\ 5$ and with $\delta = 2$.

β \ n	1	2	3	4	5	6	7	8	9	10
0	3.1227	6.2734	0.4182	12.5614	15.7040	18.8462	21.9883	25.1303	28.2721	31.4139
5	5.9364	7.2291	10.0620	13.0495	16.0968	19.1748	22.2706	25.3776	28.4923	31.6122

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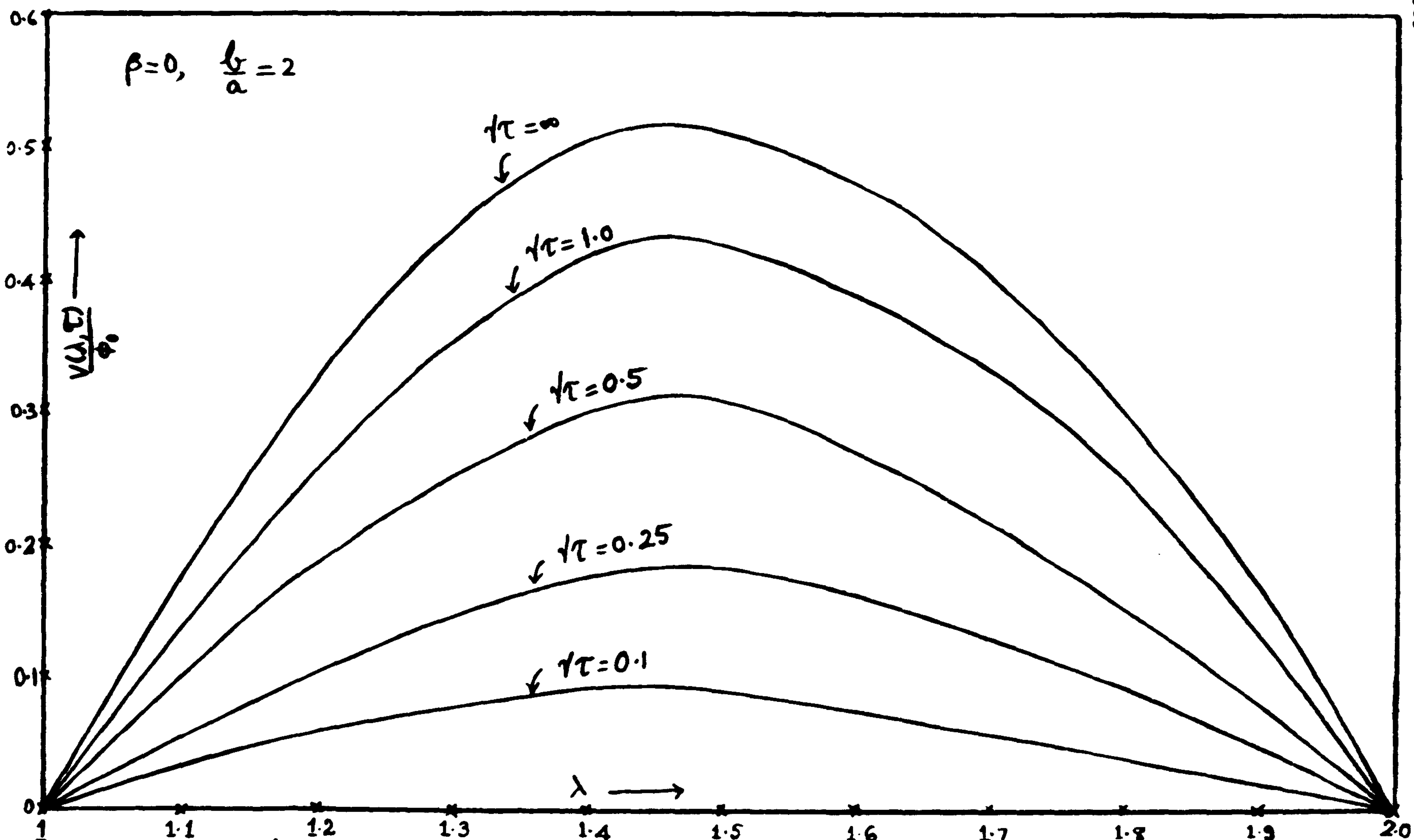
$\beta=0, \quad \frac{b}{a}=2$
$\sqrt{\tau}=\infty$
$\sqrt{\tau}=1.0$
$\sqrt{\tau}=0.5$
$\sqrt{\tau}=0.25$
$\sqrt{\tau}=0.1$
$\frac{V(\lambda,\tau)}{\phi_0}$
0.6
0.5
0.4
0.3
0.2
0.1
0
λ
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0

Fig. 2

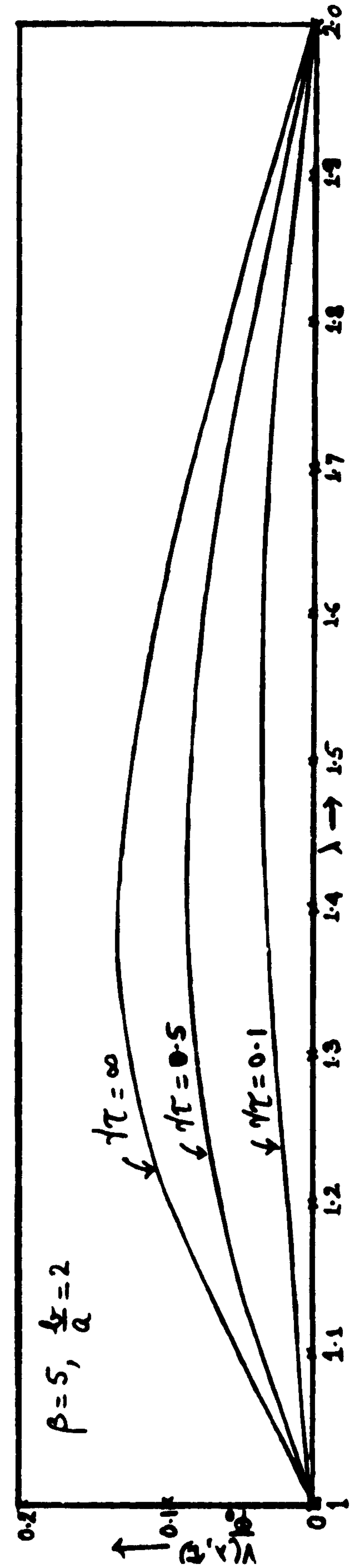

Fig. 3

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